Briefing Paper

The Socratic Methodology

Summary:

The Socratic Methodology is specifically a method of mathematics education for mixed-ability, multi-cultural, multi-lingual classes. It simply requires teachers to move progressively away from dependence on instruction towards guiding pupils to discuss the explanations in their textbooks. This approach has an initial social objective and three mathematical aims.

The social objective is to enable pupils to experience mathematics lessons as a co-operative social activity in which all can engage in critical, constructive and receptive discourse – and thus learn to give and receive correction without anger or resentment.

The first mathematical aim is to facilitate pupils' engagement with mathematics through discussion so as develop insight into, and understanding of, mathematical concepts, arguments and methods.

The second aim is to encourage pupils to solve mathematical problems on their own (as a homework exercise) since this is a way of developing understanding and problem-solving capability.

The third aim is to encourage pupils to persevere when struggling to understand a concept or solve a problem since this is a way to experience their own Eureka moments – special moments when understanding suddenly arises and the pupil perceives that which was previously hidden.

The mathematical and other benefits of using this approach are:

- a major increase in students' comprehension of mathematical concepts
- students' study and problem-solving skills are greatly improved
- critical, constructive, receptive discourse becomes natural to them; their self esteem is enhanced together with respect for their teacher and other pupils
- the majority become independent learners, well-prepared for studying science and mathematics in Higher Education or for future employment.

Background:

In its review of mathematics education in Britain in June 2008 the independent non-party policy institute *Reform* began with the statement:

'In the modern global economy, it is a combination of core techniques, flexible thinking, logic and initiative that will be critical to future success. … UK workplaces are finding themselves short of people with basic mathematical skills. Universities are being asked to select from a significantly reduced pool of applicants, a large number of whom are independently educated or from overseas.'

And concludes:

'What is needed is a concerted attempt …to change the culture in state schools. Talk about 'eliminating poverty' is futile if we do not find ways to eliminate the poverty of mind that needless condemns so many children to low grade jobs, or no jobs at all.'

A critical and constructive response:

This problem affects most Western societies in which the attempt is being made to make higher education accessible to most school-leavers.

We believe this is because school education has not adapted appropriately to this very radical change: that most schools still depend too much on teaching through various forms of instruction. This suits the upper fraction of pupils who are capable of being taught to learn independently in this way. But most pupils are not capable. As a consequence, the majority of schools are forced to lower their examination grades continually to allow them to pass.

A solution to this problem is the Socratic Methodology. This approach was developed in one of Europe's most successful international schools to prepare children from the first year of secondary to the seventh year and their final Baccalaureate examination in mathematics.

Its first aim is to prevent mathematics lessons from appearing to be individual intellectual competitions. Instead they are experienced as a co-operative *social* activity, in which all can engage in critical, constructive, receptive discourse, in which all begin to learn to give and receive correction without anger or resentment. Their self-esteem is increased, together with their respect for their teacher and for others in their class. Their shared moral values and social cohesion are strengthened. Of particularly important is the emphasis on the moral and heuristic value of personal and collective honesty.

An equally important aim is, of course, to facilitate the pupils' appreciation of the logical foundations of mathematical ideas. Ultimately, however, perhaps the most important of all for their future in mathematics, is to permit as many as possible, as early as possible, to experience their own 'Eureka moments'.

These are the private, unexpected, unpredictable moments when logical and intuitive thinking suddenly merge, and an individual suddenly knows the excitement of perceiving deeper connections and deeper truth which have previously been hidden. These moments are absolutely essential to anyone's further enjoyment of mathematics; the earlier they are experienced, the greater the appetite for more.

Pursuing all three of these aims systematically will ensure:

- a deepening of students' comprehension of essential maths concepts**;**
- that constructive discourse becomes the expected social norm;
- that their study and problem-solving skills are greatly improved;
- that the majority of pupils become independent learners, well-prepared for studying science and mathematics in Higher Education or for future employment.

This approach was studied in 1996-98 in a four-nation two-year EU *Comenius* study, directed in Germany, entitled: *'Mathematics Teaching and Democratic Education'*. More recently, in January 2009, it was the subject of a conference in Windsor Castle entitled *'Giving Peace a Voice'* funded by the Qatar Foundation and attended by participants from 12 countries.

The full report of the conference has been accepted by the Foundation.[1] In December 2008 Colin Hannaford, its organizer, won a prestigious Upton Sinclair Award for innovation in education. Previous recipients have included Her Royal Highness Queen Rania of Jordan.

[1] A full report will be found under 'New' in the website www.gardenofdemocracy.org.

The general problems:

1. Inadequate preparation: Universities find that undergraduates have mainly been taught to pass modular examinations and to maximise their grades. Many are incapable of independent learning. They often lack basic mathematical skills. They lack understanding of mathematical concepts and modelling and problem-solving skills. There is a constant need for extra remedial work - especially in mathematics - in the first university year. Further extra costs are incurred when many students abandon courses for which lecturers and technical staff have been employed.

2. Inadequate assessment: Both state and independent schools are frustrated by government directives encouraging rote learning and teaching to the test. In consequence, many independent schools are considering introducing the International Baccalaureate. This requires its students to pass at least three subjects at university entry level and two to three other subjects at an only slightly lower level. The Socratic Methodology was developed over a period of about fifteen years for students taking the Baccalaureate exam. These senior pupils consistently achieved a 100% pass rate, with an average pass mark above 70%. The major change over time, however, was their ability to continue to work alone. This is what school education should achieve.

3. Inadequate education: It has become impossible to ignore the fact that teaching mathematics to classes of mixed-ability students by instruction *simply does not work*. Students understand in different ways and at different rates. They get their 'eureka!' experience at different rates, at different times: but often not at all. There is an urgent need for a new approach to teaching mixed-ability, multi-cultural, and even multi-lingual classes. Its aim must be to enhance pupils' real mathematical understanding without training them simply to pass exams.

4. Demoralization: The declared purpose of education today is to provide nearly universal access to higher education. This change is very radical. Unfortunately the method of delivering information has not changed radically enough.

Most educational debates assume that mathematics education will succeed if only a good teacher can be placed in every class. Unfortunately, Western universities are simply not producing enough mathematics graduates for this to be possible. Even if it were, attempting to teach mixed ability classes by instruction invariably creates serious social damage. This can be confirmed by the most elementary survey.

Primary emphasis on instruction soon creates three distinct divisions in any class of young people. Each division will develop very different dominant personal traits and moralities. These divisions soon become as impenetrable as the social class systems which modern education practice was supposed to eradicate.

The first division will consist of pupils who have always understood the teacher's level of delivery of information. They will usually continue to do so. They are used to the articulation of ideas. Often this is what they hear at home. They will become personally selfish; uninterested in the welfare of the rest; sure of their right to succeed. In general, they will feel no responsibility for anyone but themselves. As adults they will tend to be politically disinterested.

The second division, usually much larger, will consist of pupils who cannot follow their teachers very well, but soon learn to obey and to copy. They also learn that their obedience may be treated by their teachers as comprehension. Within a few years, however, their early enthusiasm begins to be replaced by the recognition that this, for

them, is what their education is always going to be. Socially they become increasing alike: copying each others' manners, clothing, habits and speech. Their schools respond by testing their ability to obey and to copy, not comprehend. They learn to admire successful dishonesty and contempt for authority. As adults they will tend to believe that all success must require some degree of dishonesty, and this belief is confirmed as dishonesty and greed are repeatedly revealed in private and public affairs.

The third division have never understood much or any of their teachers' instruction. They find that they can neither obey nor copy sufficiently well to be allowed to succeed. They become increasingly angry and disruptive in trying to stop any advance by their class. As adults they will continue to feel they have been victims of society. Many will become a permanent social liability. They are the shameful consequence of societies which never adapted adequately to their needs.

Solution:

The Socratic Methodology is an inexpensive cure of this costly educational and social malaise. It may be progressively introduced into any orthodox secondary school within five years. It is not a particularly new idea. In 1945 the distinguished mathematician György Pólya wrote in his famous book*:* "It is not desirable that the teacher should present proofs in the pure Euclidean manner, *although the Euclidean presentation may be useful after a discussion in which … the students, guided by the teacher, discover the main idea of the solution as independently as possible*." [2]

Understanding demands attention. It takes effort. It requires adequate encouragement, and adequate alternatives of space and time. It also requires work.

The Method:

The aim of the teacher is to engage the attention of the class for short periods of intensive study of the explanations of mathematical topics in the textbooks which they carry to school every week and will carry home again. This is achieved by directing individual pupils to read single lines of explanation aloud directly from their textbook - and then to direct their discussion of the meaning of these lines, by the whole class together, until they all believe - with their teacher - that they have all achieved a sufficient understanding for the work ahead.

This approach, incidentally, is also a great aid to improving literacy. The weaker readers in the class are repeatedly obliged to follow a printed text whilst also hearing it read aloud by stronger readers. Then they hear its meaning discussed and explained. Since boys are often less confident and articulate than girls, this practice encourages them to be more confident and more articulate, and this can also markedly reduce any resentment that boys may feel at appearing less able than girls. In some cultures this resentment is not merely traditional: it is enforced.

Once the class has agreed on their common understanding, the pupils themselves select problems from the textbook to test their own understanding. If they cannot understand or solve the problems they have chosen, they should look again in their textbook to learn why. If there is general failure, the textbook's explanation must be collectively reviewed again to learn a better understanding. On rare occasions, teacher

[2] *'How to Solve It'* Polya, G., 1945 [revised Stewart, I. 1990 (over one million copies sold).]

and class may decide to replace an inadequate explanation with their own. In this way they learn another crucial lesson, that printed texts are not always infallible.

Consequences:

The immediate consequence of this simple change is a social and moral transformation of mathematics lessons. Children are naturally competitive. Under instruction, they are likely to attempt to prevent others from understanding better than themselves. Increasingly they will now compete instead to provide better explanations. The classroom environment is transformed from a toxic mix of jealousies, provocation and deliberate prevarication, to one in which pupils listen to one another with far more attention than ever they would usually listen to their teacher. They learn to give criticism generously and to receive it without resentment. They learn the pleasure of team success. They will understand that individual success can come much later.[3]

The broader implications:

Mathematics remains the core of modern economies. It remains the subject in which children can learn the value of personal honesty and can perceive the terrible social costs of dishonesty. In the broader context of a spiritual tradition, developing one's understanding would be viewed as a sacred endeavour, since it is central to perceiving that which is hidden and from which one is cut off. This, it may be credibly argued, is what Socrates was really most concerned about.

The more material fact is that any nation or region which falls behind in mathematics will swiftly fall behind in technology, in economy, in social harmony, in power, in all its political, economic, military and cultural dimensions.

A similar approach can be developed in many other subjects, certainly in all the sciences, also many of the humanities.

We hope that many others will follow our lead. Complacency will be fatal to any country's future

Institute for Democracy from Mathematics,
Oxford, April 2009.

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[3] 'Evaluating Change', Hannaford, C., 2006 - produced for the Qatar Foundation