Sensitivity analysis didn’t help. A practitioner’s critique of the Stern review

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1. Introduction

Exactly 20 years ago Saunders Mac Lane, a mathematician, started off a debate on the quality of quantitative modelling which, for all purposes, could have been written today (Mac Lane, 1988a,b). Then as today, the use of mathematical models in the absence of reality checks can be held responsible for a crisis of credibility in models. The antecedents of this crisis are the works of biologist Robert Rosen (Rosen, 1991) and philosopher Jean Baudrillard (Baudrillard, 1999), while the issue is popularized today by Nassim Nicholas Taleb (Taleb, 2007) in Economics, and Orrin H. Pilkey and Linda Pilkey Jarvis in Environmental Sciences (Pilkey and Pilkey-Jarvis, 2007).

Among the reality checks which modelers are request to perform when going public with their findings, sensitivity analysis (SA) plays an important role, according to existing guidelines and textbooks (OMB, 2002; OMB, 2006; EC, 2009; EPA, 2009; Kennedy, 2007; Santner, 2003; Saltelli et al., 2008).

The subject of this brief comment is the “Technical Annex to postscript” of the Stern review, which presents a sensitivity analysis (SA) addressing the conclusions of Stern review itself as well as (b) the debate on Science between Nicholas Stern and William Nordhaus. The purpose of the Stern’s Annex is to defend with a SA a cost–benefit analysis (CBA) of climate change risk performed in the Stern review. The present note calls into question the SA performed by Stern. In fact when comparing existing SA practices with the work described in the Annex, it would appear that SA has been used improperly and that – had it been used properly – it would have falsified the analysis itself. The same conclusions apply to Nordhaus’ critique in that both authors pretend to describe the issue on terms of parameters and models which bear no tested relation to reality.

The purpose of the Stern’s Annex is to defend his CBA of climate change risk in light of the received criticisms. Different objections to Stern’s review, from both ecological economics (Spash, 2007) and mainstream economics (Nordhaus, 2007; Weitzman, 2007), retrace a 30-year-old debate between the two sides on the limits to growth (Funtowicz and Ravetz, 1994; Daly, 1997; Solow, 1997; Stiglitz, 1997; Neumayer, 2007), and focus on the critical assumptions underlying the CBA framework. Thus Stern took the right approach in addressing these objections via a sensitivity analysis. SA aims to ascertain if the inference of a model-based study is robust or fragile in light of the uncertainty in the underlying assumptions. One of Edward Leamer’s most celebrated econometric work is titled “Sensitivity Analysis Would Help” (Leamer, 1990a,b), hence the title of the present work which calls into question Stern’s SA and its use to attribute a “valuable supplementary role” to the CBA (Stern, 2006; Stern and Taylor, 2007).

Comparing SA best practices as described, e.g. in EPA 2009 to the work described in the Annex, it would appear that SA has been used improperly and that – had it been used properly – it would have invalidated the analysis itself.

To a certain extent, the arguments developed in the present comment apply as well to Nordhaus (2007) who disagrees with Stern but believes that the behavior of earth climate, population and welfare in 2095 can be the subject of quantitative economic analysis, if only the discount rate could be set properly. To which Stern and Taylor retort “Our sensitivity analysis shows that our main conclusions [...] are robust to a range of assumptions.” (Stern and Taylor, 2007). A technical objection to this statement is that SA
should involve a thorough – and possibly efficient – exploration of the multi-dimensional space spanned by the input assumptions, and this is not done properly in the Annex. An epistemic critique is that SA is used to show that “an effect (in this case a high cost to society) is still present when we change the input”. Instead, the question put to SA (according to the Stern’s own plan to ensure robustness) should have been: “Is the estimate of the cost robust to the assumptions underlying the analysis?”

A key concept in Leamer’s sensitivity analysis is that of the robustness of inference. Inference’s robustness is only ensured when the input assumptions have been explored widely enough without ‘flattening’ the inference itself. The same concept is independently expressed by Funtowicz and Ravetz (1990) who warn against arbitrarily restricting the uncertainty in the input just to obtain usable results. These concepts are shared by practitioners (Kennedy, 2007; Santner, 2003; Oakley and O’ Hagan, 2004, Helton et al., 2006; Saltelli et al., 2008). Existing guidelines on impact assessment stress that sensitivity analysis should look at the entire space of the input factors and be capable to detect ‘interactions’ among factors (OMB, 2006; EC, 2009; EPA, 2009).

2. Stern’s analysis

In chapter 6 of the Stern’s report, the authors use the PAGE2002 (Policy Analysis of the Greenhouse Effect 2002) integrated assessment model to simulate the monetary costs of climate changes and how the costs rise with increasing temperatures. This is made by expressing the impacts of climate changes occurring over several decades into a single monetary metric of damage.

As any modeling exercise, the final results of the simulations are very likely to depend on some key modeling decisions related, in particular, to (i) the forecast of the temperature increase for the coming decades, (ii) the identification and quantification in monetary terms of the market and non-market impacts of climate change and (iii) the weighting of costs taking place in different regions and at different periods of time.

Given that the distributional assumptions and ethical judgements that underlie the conclusions of the analysis have been subject to criticisms, Stern and co-workers have tested through a SA the robustness of their findings. In the SA, these authors have varied the following inputs:

1. The damage exponent \( \gamma \) of the damage function. The damage function determines the relationship between the temperature and the estimated damages: the higher the damage exponent, the stronger the dependence of damages on temperatures. Two alternative distributions for \( \gamma \) both triangular, \([1, 1.3, 3]\) and \([1.5, 2.25, 3]\) are considered in the SA.

2. The pure discount rate \( \delta \). When quantifying the aggregate effect of climate changes over several decades, \( \delta \) defines the relative weight of damages happening now with those in the future. The higher \( \delta \), the lower is the present value of damages occurring in the future. Four different discount rates respectively equal to \(0.1, 0.5, 1.0 \) and \(1.5\) are used in the SA.

3. The elasticity of the marginal utility of consumption \( \eta \). This parameter is used to account for the fact that as future generations will be better off, the marginal value of their consumption decreases. The lower \( \eta \), the lower the aversion to inequality among people living in different generations with different levels of consumption, and thus the less importance we give as to whether the future is richer or poorer. Three values of \( \eta \) equal to \(1, 1.25\) and \(1.5\) are considered in the SA.

4. Climatic response to GHG emissions. The model is run under both the baseline climate \( S_b \) and high climate \( S_h \) scenarios. The high climate scenario differs in terms of higher probability of larger temperature changes.

5. Impacts covered in the CBA: the effect of climate change is estimated when (i) only the market impacts are measured, \( I_{nmi} \), and (ii) both market and non-market impacts are covered, \( I_{nmi} \).

In the SA exercise four factors are thus treated as discrete and one as continuous (\( \gamma \)). Tables PA2 and PA3 of the Annex report the mean effect of climate change in percentage losses in GDP per capita for various combinations of impact, scenario, \( \eta \), \( \delta \) and \( \gamma \). For each combination, 1000 Monte Carlo (MC) simulations have been performed, the varying parameter being \( \gamma \) for which a value has been randomly chosen from the assigned triangular distribution. This allows Stern and co-workers to present for each combination the estimated effect corresponding to the 5th and 95th percentile run. Stern’s analysis is summarized in Table 1.

For a SA practitioner Stern’s analysis rises the following questions:

1. Why is the investigation framed as from Table 1? Why not by propagating the uncertainty by the simultaneous variation of \( \gamma, \eta, \delta \), all scenarios and all impacts? Considering all combinations of scenarios, impacts, \( \eta, \delta \), and the two distributions for \( \gamma \) in Table 1, already gives 96 combinations instead of the 36 considered in the study (only 36 of the 48 cases given in Figure PA2 plus Tables PA2, PA3 of the Annex are non-repeated). How would the results look under these 96 seen simultaneously? Why not reporting the simulations which should produce the lowest effect of climate changes (high discount \( \delta \), high elasticity \( \eta \), baseline scenario \( S_b \) and impact assessment without non-market effects \( I_{nmi} \))?

2. Why is \( \gamma \) explored in a continuum and all other factors in a discrete fashion? In other words, the 5th and 95th percentiles reported in Tables PA2 and PA3 are derived from the 1000 MC simulations which differ only in terms of the value of \( \gamma \). The other four factors—varied in a discrete fashion, are fixed across the 1000 simulations, and this is inefficient.

3. Is the analysis complete? A quantitative sensitivity analysis would first characterize the uncertainty in the prediction, via an exploration of the entire space of the assumptions. This would imply an efficient, possibly non-saturated, design, followed by an estimate of the empiric distribution of the prediction (Mastrandrea and Schneider, 2004 for an example). Subsequently it would look at which assumption or combination of assumptions is mostly responsible of the variation observed in the prediction, though, e.g. a decomposition of its variance (Saltelli et al., 2005; EPA, 2009; see Chu et al., 2007 for an example and Saltelli et al., 2010 for computational aspects.).

3. A different analysis of the same data

The technical objections listed above can be summarized as ‘lack of design’. In fact this problem is not specific to the Stern review. It is common in the literature to claim that a robustness check has been achieved through sensitivity analysis, while it is very rare to find a properly designed one. Most often in climate studies – though not in Stern’s Annex – SA is reduced to changing the value of uncertain parameters one at a time (OAT) while keeping the other constant (Murphy et al., 2004; Hof et al., 2008), a practice which all practitioners and guidelines quoted thus far discourage – OAT does not explore properly and does not detect interactions among factors.

Although the SA of the Stern’s Annex cannot be replicated here – this would imply access to the models used in the analysis together with their input – we have elaborated the output from the Annex in Fig. 1(A–C).
Table 1
Factors’ combinations explored in Stern’s SA. $T(\cdot)$ indicates a triangular distribution, $S_b$ and $S_h$ refer to the baseline and high climate scenarios, while $I_{+/\text{nmi}}$, $I_{/\text{C0}}$ refer to including or not including non-market impacts (see text). Entries are the mean values and in parentheses the 5th and 95th percentiles.

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Fig. 1. Alternative analysis of Stern’s data. (A) Vertical axis: mean m (dots), 5th and 95th percentiles of % loss in GDP per capita from Stern’s analysis (Table 1). Horizontal axis: all 36 cases sorted by ascending m. (B) Cumulative distribution functions of the 36 log-normal distributions fitted to the triplets of data (mean, 5th and 95th percentiles) in (A). The root mean square error of the fit over all 36 triplets (i.e. the square root of the average of the 108 squared differences) is $\sim 0.71$. (C) Obtained by superimposing and averaging the 36 fitted log-normal distributions. Two- and three-factor Weibull distributions were also tried, but while the spread in the results remains roughly equal the log-normal distribution gives the best fit. Log-normal distributions are customarily suggested for positively skewed variates, see e.g. Helsel and Hirsch (1992).
Fig. 1(A) is simply a rearrangement of the Annex’s 36 Monte Carlo analyses, sorted by mean impact, showing mean impact and 5th and 95th percentiles for each scenario. Fig. 1(B) has been built by fitting a log-normal probability distribution (pdf) to each of the 36 data triplets (mean and percentiles) of Fig. 1(A), and plotting the resulting 36 cumulative distribution functions (cdf). Finally we have averaged the 36 pdf giving to each the same weight and produced the plot in Fig. 1(C). This figure is also a pdf (with unit area) and corresponds to treating all 36 scenarios as equally probable. This is what we would call an uncertainty analysis, answering the question ‘How uncertain or fragile is the inference?’ Note that Fig. 1(A–C)’s uncertainty analysis is predicated on all other variables and assumptions in the models being true.

These plots allow us to illustrate our dissent with Stern’s analysis as follows:

- The results from an uncertainty analysis should be displayed in a single plot, e.g. our Fig. 1(B) or better Fig. 1(C), to offer the reader a clear grasp of the overall uncertainty associated to the inference.
- It is clear from the spread of the curves in Fig. 1(B), and by the width of the curve in Fig. 1(C), that the inference is fragile. Indeed, the probability of a GDP loss below 2% is comprised between 0.017 and 0.818 depending on which curve one chooses in Fig. 1(B). Moving from the highest to the lowest cumulative distribution, we move from the beliefs of Nordhaus to that of Stern. We do not see a scientific proof of either position here.
- If one could add some of the untried extremal combinations of factors, such as highest values for δ and η, without non-market impact and for baseline climate, not reported in the Annex, this would fatten the left tail in Fig. 1(C), thus making the inference even more problematic as to the urgency or non-urgency of action to fight climate change.
- Stern’s analysis appears framed to bring the message “even if we change the assumptions we observe an important effect”. Read by a practitioner, the question to answer would be “Is the CBA robust with respect to legitimate variations in its assumptions?”, and the answer should be a qualified “No”.

Fig. 2(A–E) constitute a sensitivity analysis, answering the question ‘How can the uncertainty in the output be apportioned to different input factors?’ We have reordered the points in Fig. 1(A) by ascending value of each of the 5 factors.

Thus in all Fig. 2(A–E) the points are the same, only differently arranged. The influence pattern of each factor is now clearer than in the Annex’s original plots: the most influential factor is the one which is capable to impart to the points more variability moving along the abscissa, i.e. it is clear that including or excluding non-market impact (plot D) has more influence of the output than varying the γ distribution (plot E). One can even compute how much the mean of the ordinates changes moving along the abscissa. This is done in sensitivity analysis when using so-called variance-based sensitivity measures (Saltelli et al., 2010; EPA, 2009). The plots (A–E) show that all factors do influence the output, and confirm Stern and Taylor’s claim (2007) that scenario and impact are quite important, so that not all is driven by δ and η.

Note that a truly quantitative sensitivity analysis would need the output themselves, i.e. the 1000 points per scenario, rather than just mean and percentiles of the output. With these points one could compute a sensitivity measure built upon the input–output scatter-plots (Saltelli et al., 2000; Santner et al., 2003; EPA, 2009). Sensitivity indices could thus be computed which would allow an
unambiguous ordering of the factors by importance (see an example in Chu et al., 2007).

4. Conclusions

Our analysis of the Annex shows that the inference in the GDP loss depends dramatically on which assumption or scenario one chooses, with for instance a probability of GDP loss below 2% which varies between 0.02 and 0.8 even when omitting combinations of scenarios producing the lowest estimates, as discussed in the present paper. The Stern–Nordhaus discussion is based on a fragile – according to the Annex’s own SA – modeling exercise. Although the authors are honest in acknowledging the speculative nature of these models they still claim that these models allow exploring “the logic of the assumptions”. A practitioner’s remark would be that this tells more about the assumptions than about the world, in the absence of the reality checks mentioned at the beginning. Stern’s SA is of the ‘what-if’ type, and as such its conclusions should not be used to claim robustness of quantitative results. The entire Stern review was conducted under enormous time pressure, and this is perhaps even more true of the sensitivity analysis in the postscript, written in reaction to the criticism attracted by the review. In fact the SA was performed after the cost–benefit analysis had been made public. Kennedy offers as one of his ten commandments of applied econometrics “Thou shall confess in the presence of sensitivity”, with the relevant corollary: “Thou shall anticipate criticism” (Kennedy, 2007). A similar recommendation can be found in Saltelli et al. (2008), who suggest SA prior to publication as helpful to prepare against adversarial falsification of one’s inference. Economists and ecologists have the right to use model-based narratives. When these narratives feed into the policy process the standard of quality for models must be high, lest model use falls into disrepute (van der Sluijs, 2002) and stakeholders reject the use of models altogether, as e.g. in food policy arena (Stokstad, 2008). It would of course help if stakeholders were given a chance to test for themselves the worth of a modeling exercise. According to the OMB (2002) when models are used to uphold policy advice data and methods should be made available to the public so that the public can test for itself the impact of changing assumptions, thereby allowing a sort of extended sensitivity analysis. This would also be our position were it not for the fact that data and model availability is not always practical or effective in ensuring transparency when the matter of interest is at the cross-road of several disciplines. In this case the modeling team should do an additional effort to allow its models to be used by third parties, e.g. by making didactic, summary version of the models available to the public. Whether this is done or not we believe with Leamer (1990a,b) that in relation to sensitivity analysis “honesty is the best policy” and that modeling teams should strive run a sensitivity analysis based on best practices.

References